14

BEHAVIOUR OF SATURABLE REACTORS IN MAGNETIC AMPLIFIERS

P. N. DAS

B. E. COLLEGE, SIBPORE, HOWRAH

(*Received for publication, October 26, 1955*)

ABSTRACT. The exact behaviour of saturable reactors when used in a magnetic amplifier, depends not only on the nature of material of the core and the magnitudes of d.c. and a.c. excitations used, but it also depends, to a great extent, on the external circuit conditions. Even with an idealised core and with the optimum values of d.c. and a.c. excitations for the given magnetic amplifier, its behaviour depends on a number of other factors namely on the number of cores used, on the nature of d.c. source and the way in which a.c. and d.c. windings are connected.

Its exact behaviour in a number of different cases has been studied and explained from fundamental considerations. These cases are . 1. A single core with the d.c. source of (*i*) infinite and (*ii*) low impedance. 2. Two cores series connected with the d.c. source of (*i*) infinite and (*ii*) low impedance. 3. Two cores parallel connected with the d.c. source of (*i*) infinite and (*ii*) low impedance. The *B-H* loop described becomes unsymmetrical in certain cases but symmetrical in others. Similarly the a.c. wave becomes flat-topped in certain cases but peaky in others. The flux density and the current through the d.c. winding also vary differently in different cases. All these differences in behaviour in different cases have been explained and some important results have been deduced. The movement of the working point in the *B-H* curve with the applied a.c. voltage has been determined in different cases and it has been shown that the average value of alternating current is zero in each case. The relation of the rectified average value of alternating current to direct current has also been found out in different cases and it has been shown that the fundamental formula for amplification in the case of a magnetic amplifier has to be modified in accordance with external circuit conditions.

INTRODUCTION

A large amount of work on magnetic amplifiers has been done in the last years and many experimental results have been published ; but no satisfactory explanations of their behaviour seem to have been given. The exact behaviour of saturable reactors under simultaneous d.c. and a.c. excitations depends not only on the magnetic properties of the material of the core and on the magnitudes of d.c. and a.c. excitations, but it also depends, to a large extent, on external circuit conditions, namely the number of cores used, the nature of d.c. source and the way in which a.c. and d.c. windings are connected. The object of this paper is to find out the exact behaviour of magnetic amplifier with an ideal core and with optimum values of d.c. and a.c. excitations in different cases of external circuit conditions.

An ideal material for the core of a magnetic amplifier should have its $B-H$ curve vertical in the unsaturated region and horizontal in the saturated region and should pass abruptly from one region to the other; for then it would be possible for the a.c. winding to have a very large impedance in the unsaturated region and a very low impedance in the saturated region and, therefore it would be possible to get a very high amplification in a.c. when the working point moves from unsaturated to saturated region by the presence of a d.c. signal. In the case of some materials, like *mu*-metal, these are approximately true but we shall assume an ideal core in our investigations. The optimum value of a.c. voltage will be the maximum value which will keep the working point confined in the unsaturated region in the absence of any d.c. signal and the optimum value of d.c. signal will be that which will move the working point in to the saturated region such that it traverses both the saturated and unsaturated regions in the presence of both d.c. and a.c. excitations. We shall assume such optimum values of d.c. and a.c. excitations to be present in our investigations. Since the behaviour under steady conditions will be the same irrespective of whether we apply a.c. or d.c. first, we shall assume that d.c. is applied before a.c. for the sake of convenience of explanations, although in actual practice, d.c. is applied after a.c.

While studying the behaviour under such ideal and optimum conditions, the following fundamental points have been kept in mind.

1. When the a.c. voltage is zero in its cycle, the position of the working point while in the saturated region, is determined by the d.c. excitation applied.

2. When the a.c. voltage is zero in its cycle while the working point is in the unsaturated region, there must not be any change of flux. This will take place when the working point will reverse its path in the $B-H$ curve and so it must be in the farthest position from the saturated region when the a.c. voltage is zero.

3. If the decrease of flux produces a positive voltage, an increase of flux must produce a negative voltage. The magnitude of induced voltage either positive or negative must be equal to the rate at which the flux decreases or increases.

4. The magnitude of induced e.m.f. is given by $e = -N \dfrac{d\phi}{dt} \cdot 10^{-8}$ volts.

$$\therefore \int e \cdot dt = -N \cdot 10^{-8} \cdot \int d\phi, \quad \text{or} \int e \cdot dt = -N \cdot 10^{-8}\, \phi.$$

Thus when a change of flux ϕ takes place, the voltage point in the voltage-time curve describes an area which is equal to $N.10^{-8}\phi$.

5. The current through the a.c. winding at any instant is determined not only by the position of the working point at that moment, but also by the magnitude of d.c. excitation at the moment.

We shall consider in the present paper the behaviour in the following different cases :

1. Single core having a d.c. source of infinite impedance.
2. Single core having a d.c. source of low impedance.
3. Two cores series connected with d.c. source of infinite impedance.
4. Two cores series connected with d.c. source of low impedance.
5. Two cores parallel connected with d.c. source of infinite impedance.
6. Two cores parallel connected with d.c. source of low impedance.

The different figures and curves in diagrams I to VI give a full picture of the behaviour in the different cases respectively.

Case 1.

In this case there are two windings, one meant for a.c. and the other for d.c. signal both wound on the same core and the impedance of the signal source is infinitely large. Figures 1 and 2 in diagram 1 indicate the nature of B--H curve and of $B-H$ loop described and figures 4, 5, 6, 7, and 8 indicate the path of the working point along the B -H curve, the sinusoidal applied a.c. voltage, the nature of alternating current, the nature of flux variation, and the nature of alternating current through the d.c. winding respectively, all referred with respect to the voltage variation curve of figure 5.

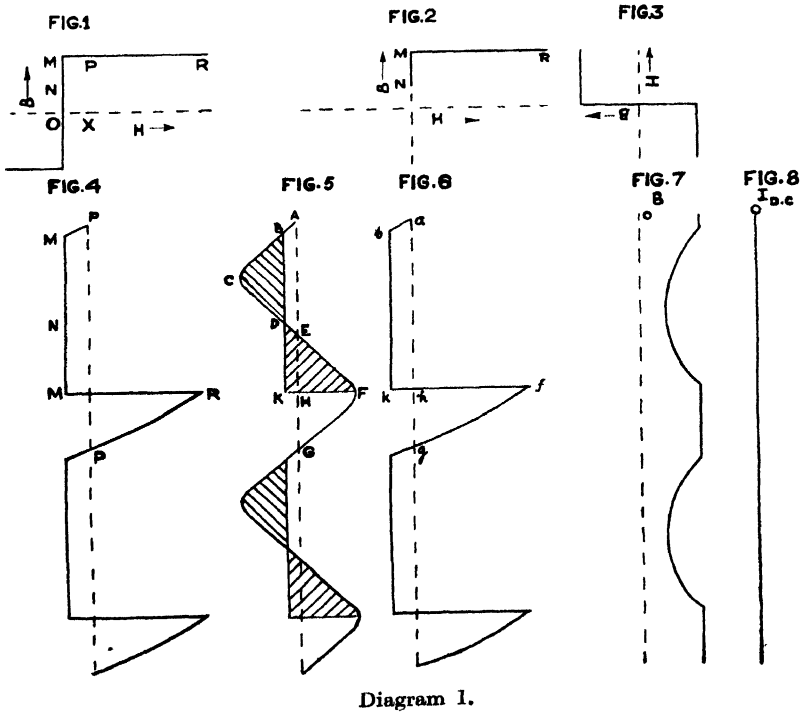


Diagram 1.

As the d.c. circuit has an infinitely large impedance, no current can flow through it by the voltage induced in it due to any flux change in the core. Therefore, the d.c. excitation OX due to the signal remains constant throughout the cycle, as shown by the dotted line in figure 4. The working point will be at P (figure 1) in the saturated region when the a.c. voltage is zero at A in the presence of d.c. signal. The working point moves from P to M as the voltage point moves from A to B say, and the current increases sinusoidally from a to b as shown in figure 6. and its magnitude at any instant is given by $E \sin \omega t / R$ where $E \sin \omega t$ is the a.c. voltage applied and R is the resistance of the a.c. winding. When the working point reaches M, it enters the unsaturated region and there can not be any further increase in current. As the voltage increases further, a change of flux takes place and the working point moves downwards in the unsaturated region at such a rate that the induced voltage due to the change of flux, will balance the increase in applied voltage. The working point will go on moving downwards and the change of flux will continue to take place in the same direction till the voltage point reaches D where its magnitude is the same as that at B. The area of the voltage-time curve between the points B and D, as shown by the hatched portion, is proportional to the total change of flux taking place when the working point moves from M to N. As the voltage point moves beyond D, it becomes positive with respect to the point D and so to balance the voltage now, there must be change of flux in the opposite direction and so the working point moves upwards from N towards M. The working point will come back to M when the voltage point reaches F where the voltage time area between points D and F, as shown by the hatched portion, is equal to the former hatched area. When the point M is reached, there can not be any further change of flux. So the working point suddently moves from M to R in the saturated region and the current, which was so long constant and negative, suddently become positive and reaches such a value that this current multiplied by the resistance R becomes equal to the voltage at F. After that, the working point being in the saturated region, the current varies sinusoidally with the voltage and the working point comes back to P and so the current is zero when the voltage becomes zero at G. This is repeated similarly in subsequent cycles. The current curve is shown by *abkhfg* in figure 6, and the nature of variation of B by the curve in figure 7. The $B-H$ loop described will be as shown in figure 2.

In voltage-time curve, the area BCD = the area DKF. So the area $ABKH$ = the area HFG. It follows, therefore, that in current-time curve in figure 6, the area *abkh* is equal to the area *hfg*. Therefore, the average value of alternating current is zero. The average value of rectified alternating current will be given by the average value of height of either of these two areas. The average value of height from area *abkh* is given by $I_a(\mathrm{av})/N_a = OX$.

But $I_c N_e = OX$. $\therefore$ $I_a(\mathrm{av}) = I_c N_c / N_a$.

There is hardly any such practical case in which the signal source produces a current due to the signal, but at the same time offers an infinitely large impedance to the voltage induced in the d.c. winding due to change of flux. A practical case arises when there is no d.c. winding at all and the d.c. excitation due to the signal is applied from a large current passing through a single conductor or when the d.c. excitation is furnished by a magnet.

Case 2

In this case the impedance of the d.c. source is low, so whenever there will be any change of flux in the core, the voltage that is induced in the d.c. winding, will produce a current through it and so the d.c. excitation furnished by the d.c. signal will change, as shown by the dotted curve in figure 4, in diagram II in such a direction that it would tend to oppose the change of flux due to which it is produced. This happens when the working point enters unsaturated region at

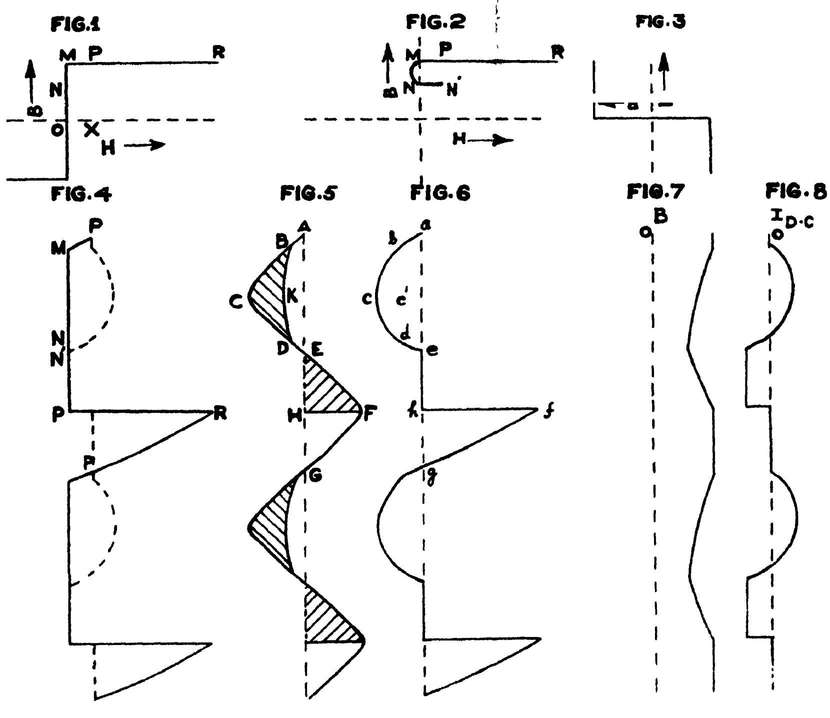


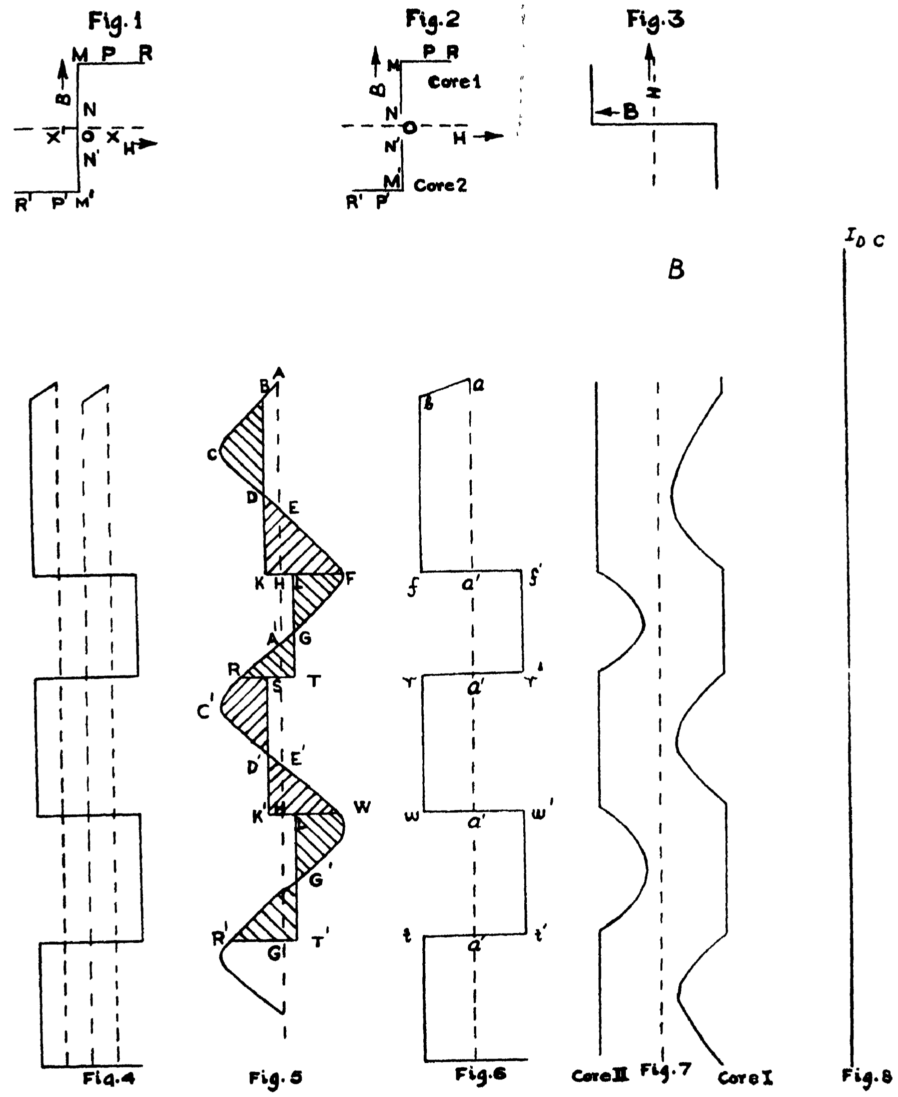
Diagram II.

M (figure 1) and a larger current flows through the a.c. winding which produces larger ohmic drop and so a lesser change of flux is necessary to balance the lesser voltage time area as shown by the hatched portion *BCDK* (figure 5) than in the former case. When the voltage point reaches *D*, the working point reaches the extreme end of its path *N*. As the voltage point moves beyond *D*, it becomes positive with respect to *D* and a change of flux in opposite direction is necessary to balance the voltage. But any change of flux in this direction will produce a current through the d.c. winding in the opposite direction and it will affect the d.c. excitation in the opposite direction till it becomes zero. When the total d.c. excitation becomes zero, there can not be any further change in it; as it would now act as a transformer whose secondary is short-circuited and there will be appreciable change of flux to balance out the voltage. This change of flux in the opposite direction may be assumed to take place practically when the voltage is zero at E, as the change of flux necessary to bring the total d.c. excitation to zero, is small. The working point will come back to M when the voltage point reaches F where the voltage time area EFH is equal to the area $BCDK$. When the voltage point reaches E from D, the resultant d.c. excitation becomes zero and therefore, although the working point remains fixed at N, the current through the a.c. winding becomes zero. In the $B-H$ loop (figure 2.) it will appear as if the working point has moved from N to N'. When the working point moves from N to M (figure 1), the current is still zero due to the resultant d.c. excitation being zero and it will appear, therefore, that the working point moves along $N'P$ in figure 2. The behaviour in the saturated region will be same as in case 1. The whole operation will be repeated in subsequent cycles. Thus the nature of $B-H$ loop described will be as shown by $NN'PRPM$ in figure 2. and the nature of current variation is shown by *abcde* and *hfg* in the two half cycles. As the area $BCDK$ = the area EFH, the area $ABKDE$ = the area HFG in the voltage-time curve. Therefore, the resultant alternating current is zero. The average value of rectified current will, therefore, be the average value from area *abcde*. It is clear that the value of this area will depend on the impedance and number of turns of the d.c. winding circuit. This area will be greater for lower impedance and larger number of turns of the d.c. circuit. If this area be equal to the area *abc'de*, then the average value will be given by I_a (av). $N_a = OX$. But $OX = I_c.N_c$. $\therefore$ $I_a(\text{av}) = I_c.N_c/N_a$. As this area is always greater than the area *abc'de*, I_a is always greater than $I_c.N_c/N_a$. So I_a/I_c is always greater than N_c/N_a. The nature of variation of B and of current due to flux change through the d.c. winding are shown in figsure 7 and 8 respectively and the $B-H$ loop traced will be as shown by $NN'PRPM$ of figure 2.

CASE 3

When two cores are series connected, the a.c. windings are connected in series and the d.c. windings are connected in series opposition. The object of connecting the d.c. windings in series opposition is that when flux changes in both the cores at the same rates, the resultant induced e.m.f. in the d.c. windings is zero. The effect of connecting two a.c. windings in series is that the same current must always flow through both the windings and, therefore, the current through any winding can not abruptly increase to a very high value when one of the cores becomes saturated. When flux changes in one core only, the voltage is induced in its d.c. winding only and though it can not be balanced by the voltage in the d.c. winding of the other core, it can not produce any current through the d.c. winding due to the infinitely large impedance of the d.c. source. Therefore, the d.c. excitation remains constant throughout as shown by the dotted line in figure 4 in diagram III.

In figure 1, $ONMPR$ and $ON'M'P'R'$ are the $B-H$ curves for the two cores. As d.c. windings are oppositely connected, current through them will produce d.c. excitation, as given by OX and OX' in the two cores. When the working point in core I reaches M as the voltage point reaches B from A, the same in core II reaches R'. When the working point in core I, has reached M, there will be



gram III.

change of flux in it before current can increase any further; and so the working point in core II, although in saturated region, can not move beyond R' when the voltage increases. The voltage is entirely balanced by the induced e.m.f. in core I by change of flux in it and the current remains constant. This change of flux continues until the voltage point raches D where the voltage is of the same magnitude as at B. When the voltage point moves further, it becomes positive with respect to D and so change of flux begins to take place in the opposite direction. This change of flux in opposite direction continues to take place till the working point in core I, reaches back to M when the voltage point reaches F such that the area BCD = the area DFK. Now there can not be any further change of flux in core I, and so the working points in both the cores will move abruptly from M to R and from R' to M' in the saturated regions respectively and current point will move suddently from f to f', as shown in figure 6. This current can balance a portion HL of the voltage and to balance the remaining portion of voltage, there must be a change of flux in core II as the working point in core II has reached M' in the unsaturated region and there can not be any further change of current. This change of flux continues to take place till the voltage point reaches G where its magnitude is same as that at L. As the voltage point moves beyond G, it becomes negative with respect to G and so the working point in core II must move in the opposite direction to produce an opposite change of flux. This change of flux continues till the working point in core II reaches M' at the voltage point R such that the area LFG = the area GRT. Throughout this time the current remains constant at the same previous value but in the opposite direction. When the working point reaches M' in core II, the working points will move again from R to M and from M' to R' and the new current will balance a portion of voltage, the remaining portion being balanced by change of flux produced this time by core I.

It follows that the area $a'rwa'$ = the area $a'w't'a'$ in the current-time curve in figure 6. Therefore, the resultant current through the a.c. winding is zero and the average value of rectified current is given by $I_a(\text{av}).N_a = OX = I_c.N_c$. $\therefore$ $I_a(\text{av}) = I_c.N_c/N_a$. Figure 7 shows the variations of flux in the two cores and figure 2, shows the nature of $B-H$ loops in the two cores.

Case 4

This is the most important case so far as practical magnetic amplifiers are concerned. This is similar to case 3 except that the d.c. source is of low impedance; so the resultant induced voltage across the d.c. windings must be zero. Therefore, $N_c\left(\frac{d\phi_1}{dt}\right) + N_c\left(\frac{d\phi_2}{dt}\right)$ must be zero and $\left(\frac{d\phi_1}{dt}\right) = -\left(\frac{d\phi_2}{dt}\right)$. So if there be any change of flux in one core, there must also be a simultaneous change in the other core under steady conditions and the rate of change must also be the same in both the cores when number of turns is the same in them. If simultaneous flux change is not possible initially in both the cores due to one core being in the saturated region, a large current will flow through the d.c. winding due to flux change in one core and this will adjust conditions in such a way that flux will change simultaneously in both the cores under steady conditions.

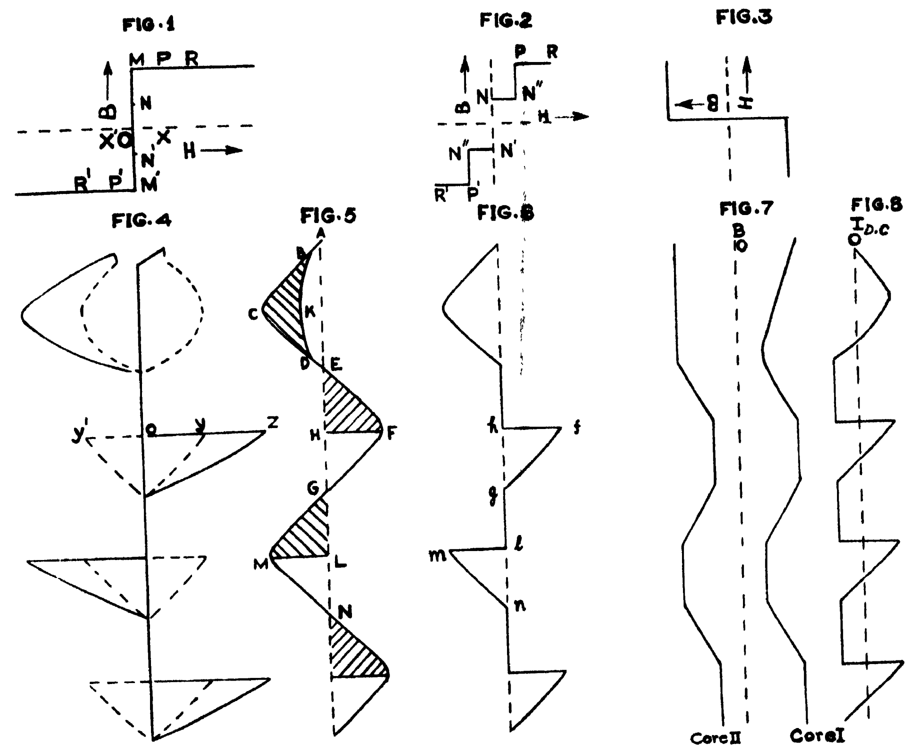


Diagram IV.

In this case the change of flux that takes place in core I after the voltage point goes from A to B, induces a voltage in the d.c. winding of core I which produces a current through it and the d.c. excitation varies as shown by the dotted lines in figure 4, in diagram IV and so the current through the a.c. winding increases and the change of flux necessary to balance the increase in voltage becomes smaller as the area of voltage-time curve which has to be balanced, becomes smaller and equal to $BCDK$ as shown in figure 5. This change of flux continues to take place until the voltage point is at D where the voltage is of same magnitude as at B. As the voltage point goes beyond D, it becomes positive with respect to D and a change of flux in opposite direction is necessary to balance the voltage. But any tendency to change the flux in this direction will produce a current through the d.c. winding in the opposite direction, and the resultant d.c. excitation will change till it will become zero in both the cores. This will take place just after the voltage point crosses D before there is any appreciable change ot flux depending on the d.c. winding and the d.c. source. As the change of flux necessary is negligible, the working points will be practically at N and M' when the voltage point reaches zero at E; but as d.c. excitation has decreased, it will appear that the working point in core 1 has moved from N to N'' (figure 2). As the voltage increases now, the cores will behave as having their secondaries shorted and there will be change of flux in both the cores at the same rate till the working point in core I reaches saturation region at M when the voltage point is at F such that the area EFH = twice the area $BCDK$. There can not be any further change of flux as core I reaches saturation and also the working point at M can not move horizontally as such, since the working point in core II is at N' in the unsaturated region. But the voltage at F can not be balanced as such and will tend to produce a change of flux in core II. But this tendency to produce change of flux will produce a current in the d.c. winding and the d.c. excitations in the two cores will change from O to OY and OY' respectively. So, although the working point in core II remains at N', a current flows through its winding and the working point in core I moves to Z (figure 4) such that the same current flows through its winding also. The d.c. excitations change in such a way that the current corresponding to OY' or YZ, can balance the voltage at F. As the voltage point goes beyond F, the d.c. excitations change in such a way that the current balances the voltage always till the voltage point comes to zero at G when the total d.c. excitation in the two cores will be again zero. The working point in core I reaches M again and that in core II remains at N'. As the voltage point goes beyond G, there is again change of flux in the cores till the working point reaches M' in the saturated region in core II when the voltage point reaches M such that the area GML = the area EFH. The same operation is repeated in subsequent cycles. It is clear that the average value of current in each half cycle is same and so the resultant current in zero. The average value of rectified current is proportional to the average value of area hfg or lmn in current curve of figure 6. Now the area HFG in the voltage-time curve = the area $ABCDE$ – twice the area $BCDK$ = the area $ABKDE$ – the area $BCDK$ and if this is equal to the area $ABDE$, then $I_a(\text{av}) = I_c.N_c/N_a$ as before. This happens when the area $BCDK$ = the area BKD. If the impedance of d.c. circuit is not very low, the area $BCDK$ will be greater than the area BKD and $I_a(\text{av})$ will be less than $I_c.N_c/N_a$. Figure 7 shows the variation of flux in both the cores and it is clear that change of flux takes place simultaneously in them. Figure 8 shows the nature of current through the d.c. winding due to hange of flux and figure 2 gives the nature of B–H loops described as $NN''PR$ and $R'P'N'''N'$ in two cores.

CASE 5

In this case the two a.c. windings are connected in parallel and the impedance of the d.c. source is infinitely large. So, though there may be a voltage induced in the d.c. windings when there is change of flux in one core only, no current can flow through it due to very high impedance of the d.c. source. Therefore, the d.c. excitations in the two cores will remain constant throughout. It is just equivalent to two single cores with d.c. source of infinitely large impedance (case 1) connected in parallel. The current through the a.c. winding of any core in one half cycle will be just the image opposite of the current in the winding of the other core in the other half cycle. Thus curve *abcdef* (figure 6) in diagram V gives the nature of current through one winding and the curve *a'b'c'd'e'f'* gives the nature of current through the other winding. The resultant current

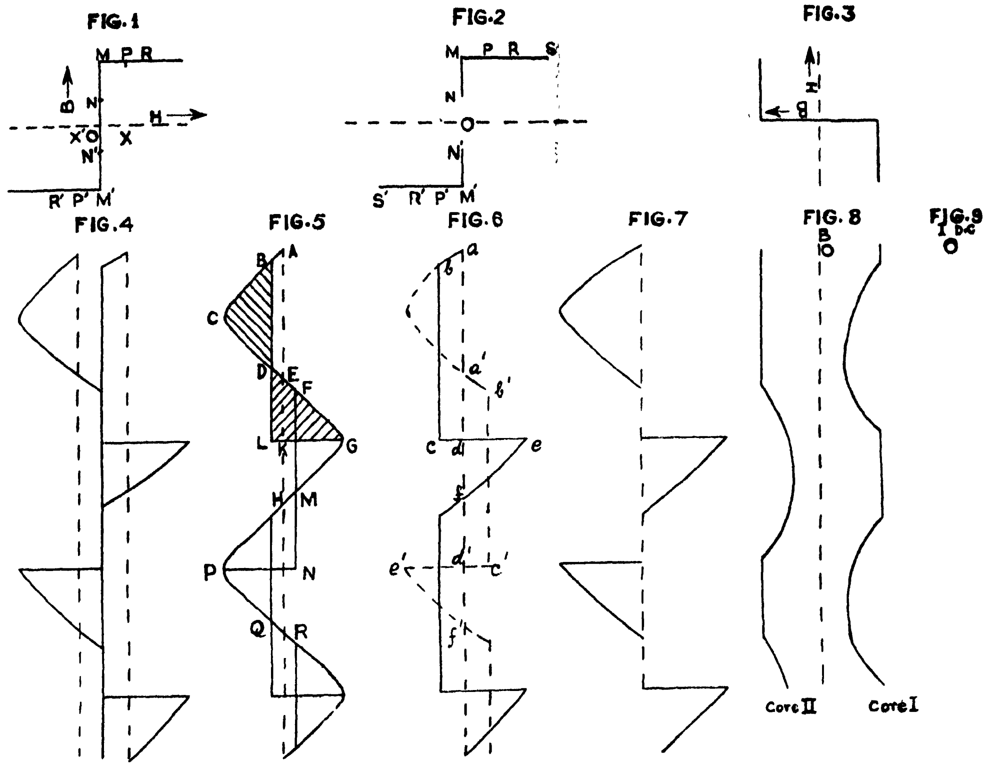


Diagram V.

which will be the sum of the currents through the two cores is shown in figure 7. It is clear that the resultant current will be zero and the average value of rectified current will be double the value given by a single core with d.c. source of infinitely large impedance. Therefore, the average value is given by $I_a(\text{av}) = 2\, I_c.N_c/N_a$. The nature of variation of B is shown by figure 8 and the nature of $B-H$ loops in the two cores will be as shown by $NMPRS$ and $N'M'P'R'S'$ in figure 2.

CASE 6

In this case the a.c. windings are connected in parallel and the d.c. source is of low impedance. When the working point in core I enters the unsaturated region with the increase of voltage, there is change of flux in it as before, but current in core II goes on increasing as shown in figure 6. in diagram VI, the working point being in the saturated region. The change of flux in core I induces

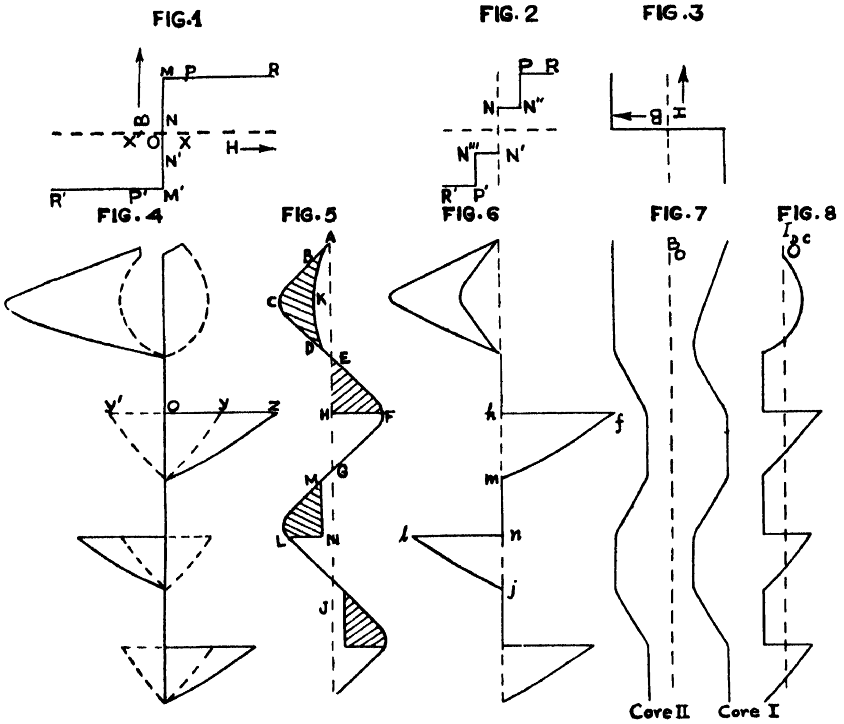


Diagram VI.

a voltage in its d.c. winding and a current flows through it which changes the d.c. excitations in the two cores as shown by the dotted lines in figure 4. This change of flux continues until the voltage point reaches D where the voltage is of the same value as at B. Then there will be a flux change in core I in the opposite direction to balance the voltage which becomes positive. As the d.c. winding is of low impedance, a current flows through it in opposite direction before any appreciable change of flux takes place and the resultant d.c. excitations in the two cores becomes zero when the voltage is zero at E, as in previous cases. So when the voltage is zero at E, the working points will be at N and M' and there will be change of flux in both the cores to balance out the applied voltage. When the working point in core I reaches at M at the voltage point F such that the area EFH = the area BCD (figure 5), there can not be any further change of flux in core I. The working point in core I will move to Z in the saturated region and there will be abrupt increase of current to balance the applied voltage at F in the core winding. In core II there will be a tendency to undergo a further change of flux to balance out the voltage and this will induce a voltage in the d.c. winding which can not be opposed by a similar voltage in the other d.c. winding and so a direct current will flow and the d.c. excitations will be changed, as shown by the dotted lines. The d.c. excitations will be changed in such a way that the currents in the two cores which will be given by YZ and $Y'O$, will individually balance the voltage at F. As the voltage changes, the working point in core I moves along the saturated region and that in core II remains fixed at N' and the d.c. excitations vary in such a way that the currents balance the voltage at each point till the voltage point is at M when the working point in core I reaches M (figure 1). Now there will be change of flux in both the cores to balance the applied voltage till the working point in core II reaches M' at the voltage point L where the area MLN = the area EHF. The working point in core II moves in the saturated region and that in core I remains fixed at N and the d.c. excitations change as before and the cycle is repeated.

The current I_1 or I_2 will be practically equal and their average values will be proportional to the area HFG. Therefore, their resultant current which will be I_1+I_2, will be proportional to twice the area HFG. But the area HFG = the area $ABKDE$. Therefore, it is clear that the average value of current will always be greater than $I_c.N_c/N_a$ and the amount by which it will be greater, is given by the area BKD. The lower the impedance of the d.c. source, the greater will be this area and hence greater will be the amplification. The nature of variation of B and of current through the d.c. winding due to change of flux are given by figures 7 and 8 and the nature of $B-H$ loops described, are given by $NN''PR$ and $N'N'''P'R'$ in figure 2.

EXPERIMENTAL OBSERVATIONS AND CONCLUSION

The behaviour of the saturable reactor under above conditions was experimentally verified with the help of a C.R.O. tube and was found to be very much like the cases under idealised conditions as given above. As mentioned above, it is very difficult to get a d.c. source of infinite impedance. An approximate condition was reached by connecting a choke of high value in series with the d.c. source of high value. Two ring cores of *mu*-metal, each having 500 a.c. turns and 1,550 d.c. turns, were used. In the case of a single core with d.c. source of low impedance, the current amplification was found to be approximately equal to 7; but in the case of two cores series connected with the d.c. source of low impedance, it was found to be of the order of 3.8. The value of B was measured by means of an integrating circuit connected in a seperate winding.

Therefore, it is clear that when the d.c. source is of infinitely large impedance which is a very rare case, we can use a single core and get the same amplification and hence the same advantage as two cores connected in series. If they are connected in parallel, however, we can get the double amplification. If the source is of very low impedance, although a single core can give the same amplification as two cores, a single core can not be used in practice due to the fact that, generally, when the d.c. signal is absent, the low impedance connected to the d.c. winding is still present and this produces a d.c. excitation even in the absence of the d.c. signal. Under these circumstances, therefore, the two cores with either series or parallel connection are used. From the point of consideration of low value of time constant, series connection is, however, more frequently used than the parallel connection.

ACKNOWLEDGMENTS

The author gratefully acknowledges the invaluable guidance and help received from Dr. H. Rakshit, Head of the Department of Physics and Communication, Bengal Engineering College, Sibpore.

REFERENCES

Miles, J. G., 1951. 'Bibliography of Magnetic Amplifier Devices and the Saturable Reactor Art,' Proc. A. I. E. E., 70.

Say, M. G., 'Magnetic Amplifiers and Saturable Reactors', Newnes Electrical Engineering Progress Series, London.

Geyger, W. A. 'Magnetic Amplifier Circuits', McGraw-Hill Book Co. Inc.. New York, N.Y.